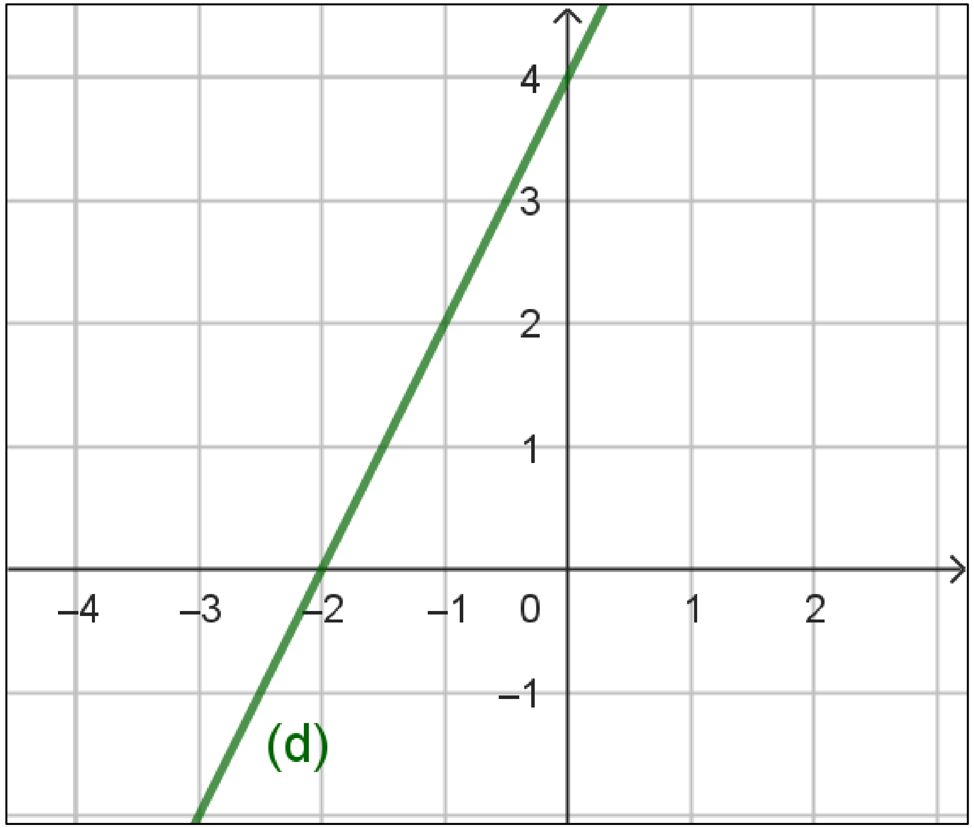
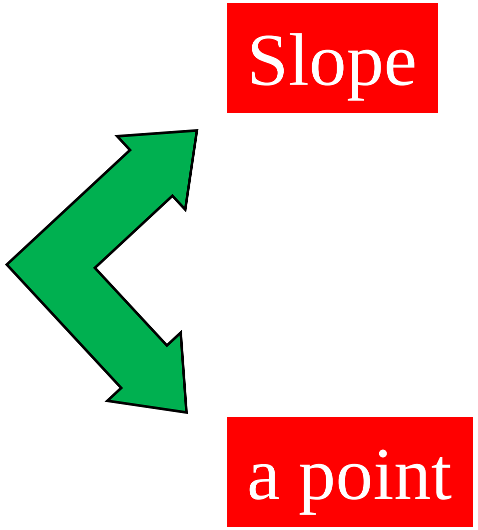
Coordinates System

Part 2

☞Equation of a line



To find the equation of a line:



☞Equation of a line

Case ①

Equation of a line of slope a and passes through a point A($x_A; y_A$):

General form: $y - y_A = a(x - x_A)$

Example:

$a = 2 \quad ; \quad A(1;2)$

$y - y_A = a(x - x_A)$

$y - 2 = 2(x - 1)$

$y - 2 = 2x - 2$

$y = 2x - 2 + 2$

$y = 2x$

☞Equation of a line

Case ②

Equation of a line that passes through points A($x_A; y_A$) and B($x_B; y_B$):

Step 1: Calculate the slope: $a = \frac{y_A - y_B}{x_A - x_B}$

Step 2: General form: $y - y_A = a(x - x_A)$ or $y - y_B = a(x - x_B)$

Example:

$A(1;2)$; B(3;4)

$$a = \frac{y_A - y_B}{x_A - x_B} = \frac{2-4}{1-3} = -\frac{2}{-2} = 1$$

$$y - y_A = a(x - x_A)$$

$$y - 2 = 1(x - 1)$$

$$y - 2 = x - 1$$

$y = x - 1 + 2$ so $y = x + 1$

☞Equation of a line

Case ③

Equation of a line that passes through points A($x_A; y_A$) and parallel to a line (d’):

Step 1: Find the slope: $a = a_{(d')}$

General form: $y - y_A = a(x - x_A)$

Example:

$A(1; 2)$; (d’): $y = 2x + 3$

$a = a_{(d')} = 2$

$y - y_A = a(x - x_A)$

$y - 2 = 2(x - 1)$

$y - 2 = 2x - 2$

$y = 2x - 2 + 2$ so $y = 2x$

☞Equation of a line

Case ④

Equation of a line that passes through points A(x_A; y_A) and perpendicular to a line (d'):

Step 1: Find the slope: $a \times a_{(d')} = -1$; $a = -\frac{1}{a_{(d')}}$

General form: $y - y_A = a(x - x_A)$

Example:

$A(1;2)$; (d'): $y = 2x + 3$

$a \times a_{(d')} = -1$; $a \times 2 = -1$; $a = -\frac{1}{2}$

$y - y_A = a(x - x_A)$

$y - 2 = -\frac{1}{2}(x - 1)$

$y - 2 = \frac{1}{2}x + \frac{1}{2}$

$y = \frac{1}{2}x + \frac{1}{2} + 2$

$y = \frac{1}{2}x + \frac{5}{2}$

☞Equation of a line (Particular cases)

Case ①

If the line passes through the origin, then the equation is $y = ax$

Example 1:

slope $a = 2$ and passes through O

The equation is $y = ax$

So $y = 2x$

Example 2:

Equation of (AO) where A(1;2)

The equation is $y = ax$

$$a = \frac{y_A}{x_A} = \frac{2}{1} = 2$$

So the equation is $y = 2x$

☞Equation of a line (Particular cases)

Case ②

If the line passes through A and B where $x_A = x_B$

$x_A = x_B = k$

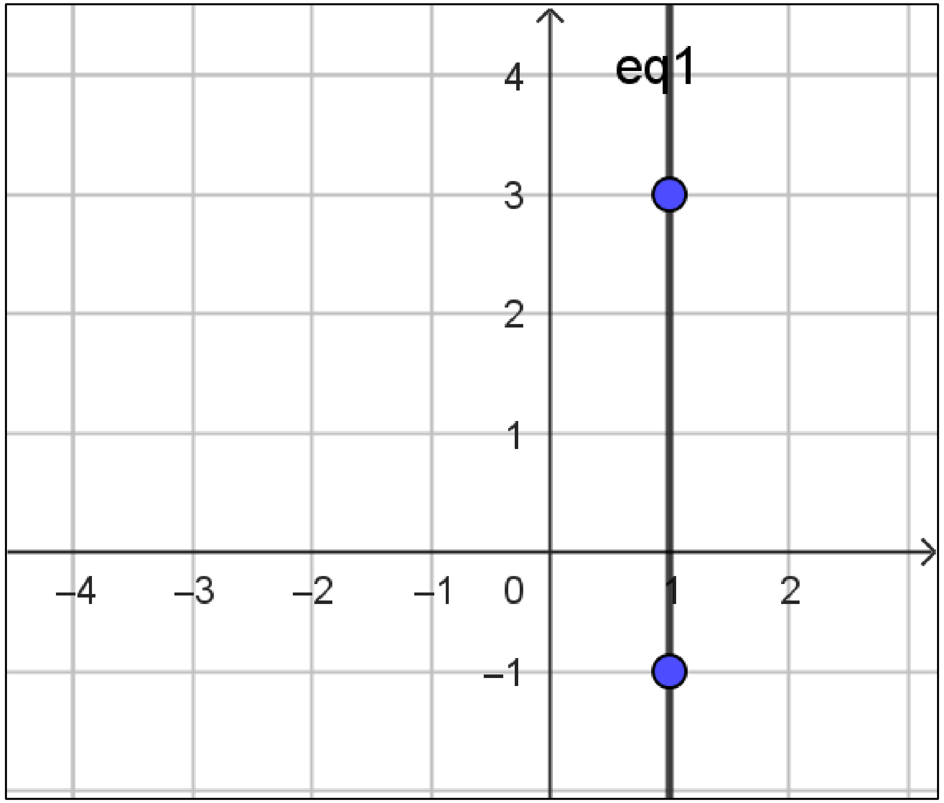
So the line is parallel to (y'y) and of equation $x = k$

Example :

A(1;3) and B(1;-1)

$x_A = x_B = 1$

So the equation is $x = 1$



Remark:

The equation of $(y'y)$ is $x = 0$

☞Equation of a line (Particular cases)

Case 3

If the line passes through A and B where $y_A = y_B$

$y_A = y_B = k$

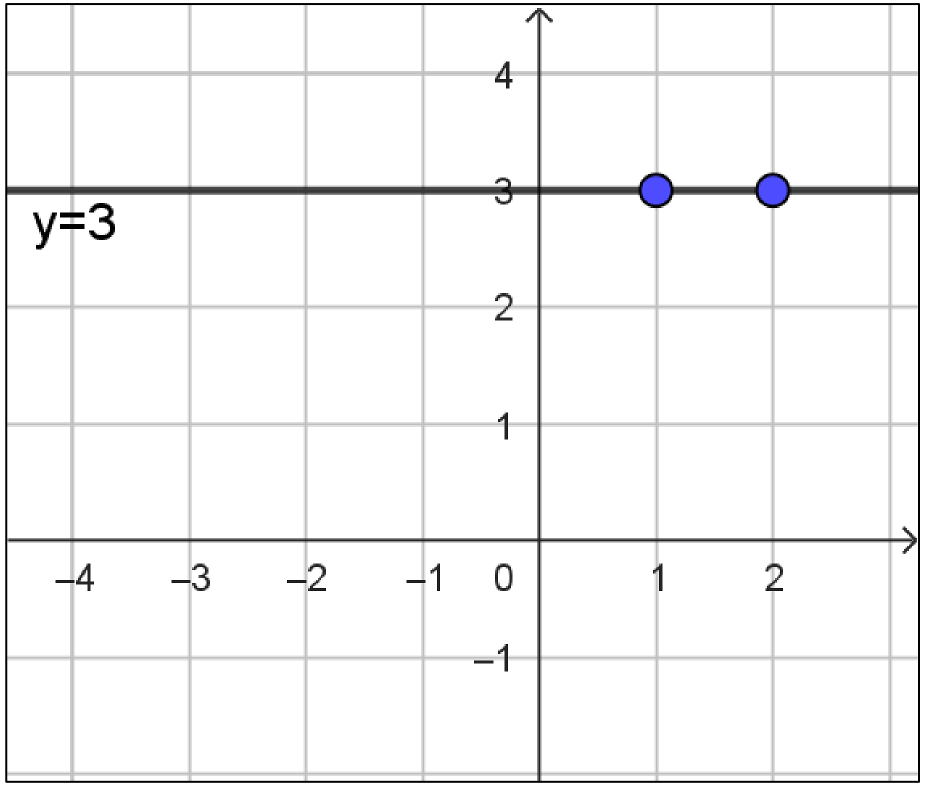
So the line is parallel to (x’x) and of equation $y = k$

Example :

A(1;3) and B(2;3)

$y_A = y_B = 3$

So the equation is y= 3



Remark:

The equation of $(x'x)$ is y $= 0$

☞Application #1

Find the equation of the line (d) in each case:

❶ (d) passes through the origin and parallel to the line (d') $y = -x + 2$.

General form: $y = ax$

(d)//(d') so $a = a_{(d')} = -1$

So the equation of (d) is $y = -x$

☞Application #1

Find the equation of the line (d) in each case:

❷ (d) passes through the points A(-1;3) and B(-2;0).

General form: $y - y_B = a(x - x_B)$

$$a = \frac{y_A - y_B}{x_A - x_B} = \frac{3-0}{-1-(-2)} = \frac{3}{-1+2} = 3$$

$$y - 0 = 3(x + 2)$$

$$y = 3x + 6$$

☞ Application #1

Find the equation of the line (d) in each case:

❸ (d) is perpendicular to $(x'x)$ and passes through A(-1;3)

(d) ⊥ $(x'x)$

So, (d) // $(y'y)$

Then the equation of (d) is $x = k$

(d) Passes through A so $x_A = k$; $-1 = k$

So the equation of (d) is $x = -1$

☞ Application #1

Find the equation of the line (d) in each case:

❹ (d) is the perpendicular bisector of [AB] where A(1;1) and B(-1;3)

(d) ⊥ (AB) so $a \times a_{(AB)} = -1$

$a_{(AB)} = \frac{y_A - y_B}{x_A - x_B} = \frac{1-3}{1-(-1)} = -\frac{2}{2} = -1$

So $a = -\frac{1}{a_{(AB)}} = \frac{-1}{-1} = 1$

(d) Passes through M the midpoint of [AB].

$x_M = \frac{x_A + x_B}{2} = \frac{1-1}{2} = 0$ and $y_M = \frac{y_A + y_B}{2} = \frac{1+3}{2} = \frac{4}{2} = 2$

$y - y_M = a(x - x_M)$

$y - 2 = 1(x - 0)$

$y - 2 = x$

$y = x + 2$

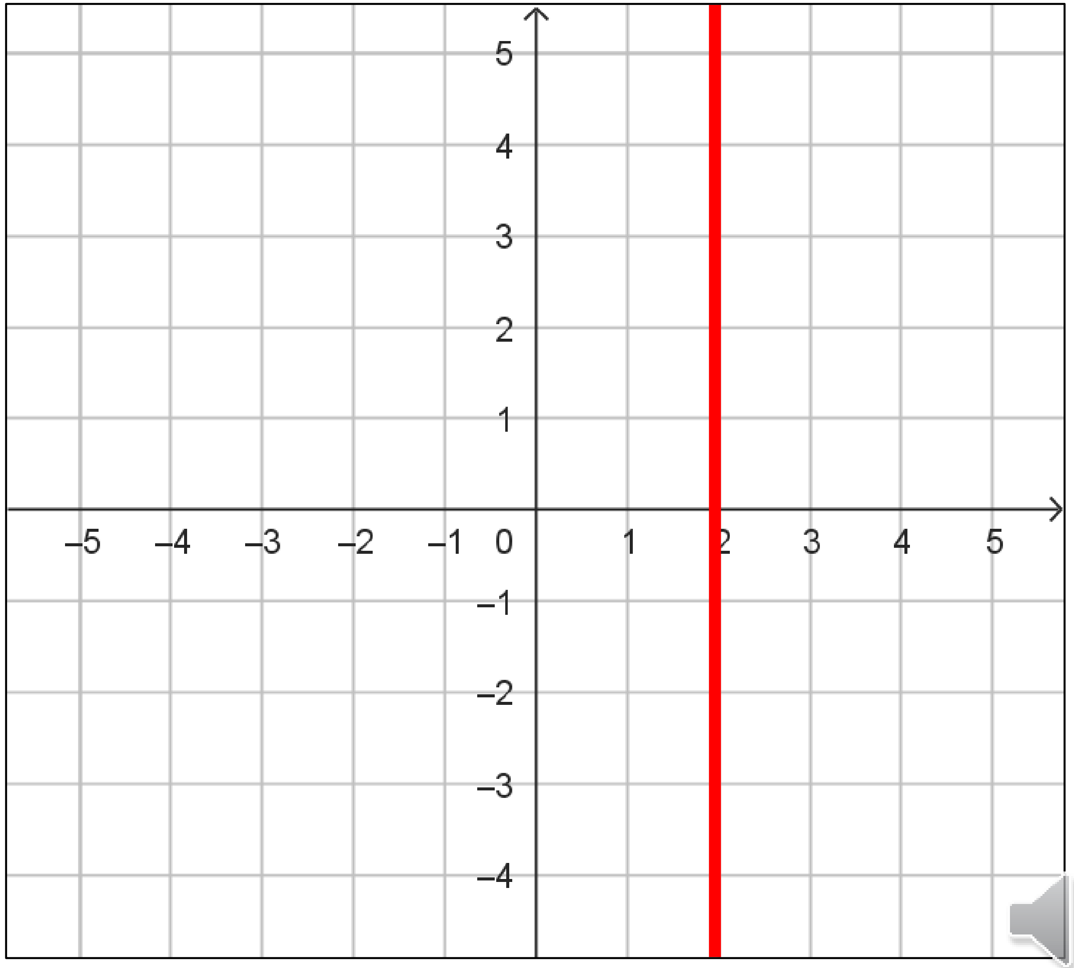
☞Plotting a line

❶ Plot a line of equation $x = k$.

Just draw a line parallel to (y’y), perpendicular to (x’x), at the corresponding abscissa.

Example:

Draw the line of equation $x = 2$.



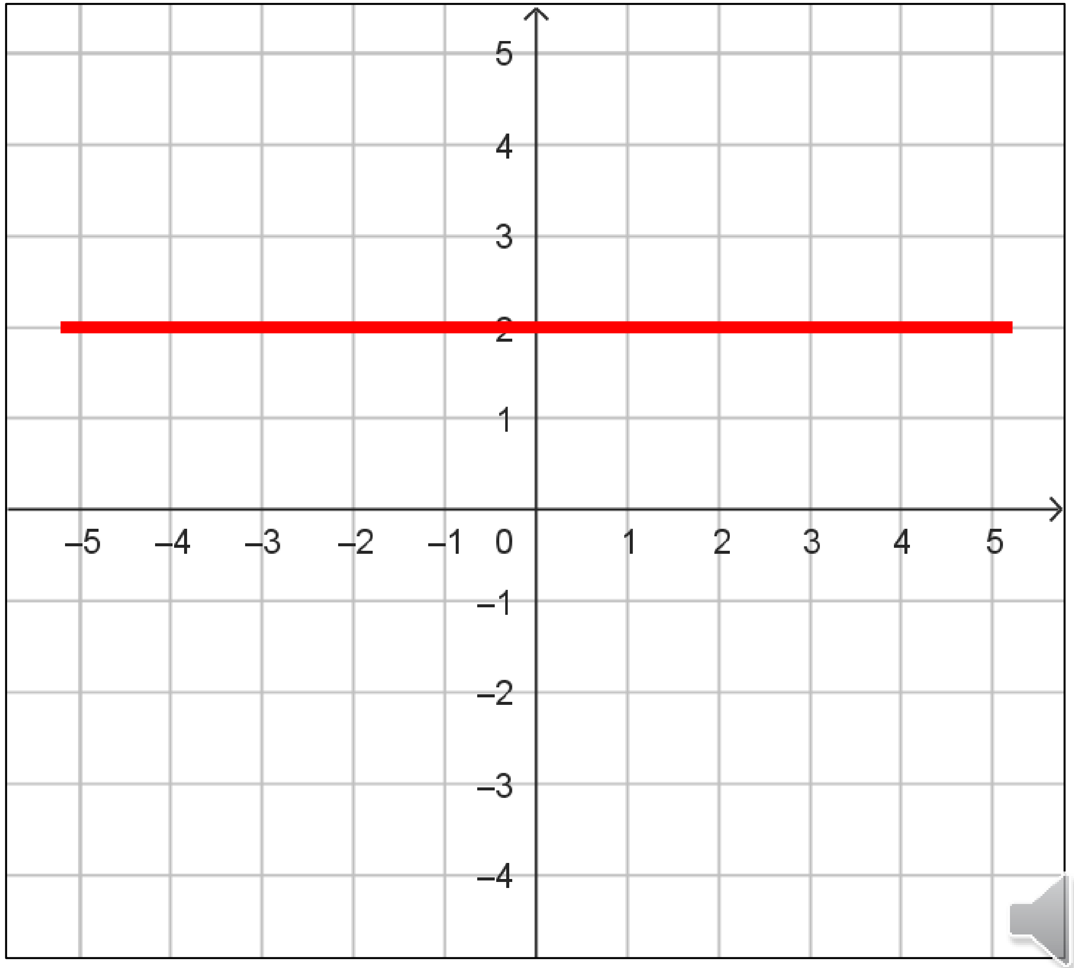
☞Plotting a line

❷ Plot a line of equation $y = k$.

Just draw a line parallel to (x’x), perpendicular to (y’y), at the corresponding ordinate.

Example:

Draw the line of equation y= 2.



☞Plotting a line

❸ Plot a line in the form of $y = ax + b$

2 points are needed:

Take two values of x (of y) and substitute each one in the equation of the line to get the ordinates (abscissas) of the two points.

Example:

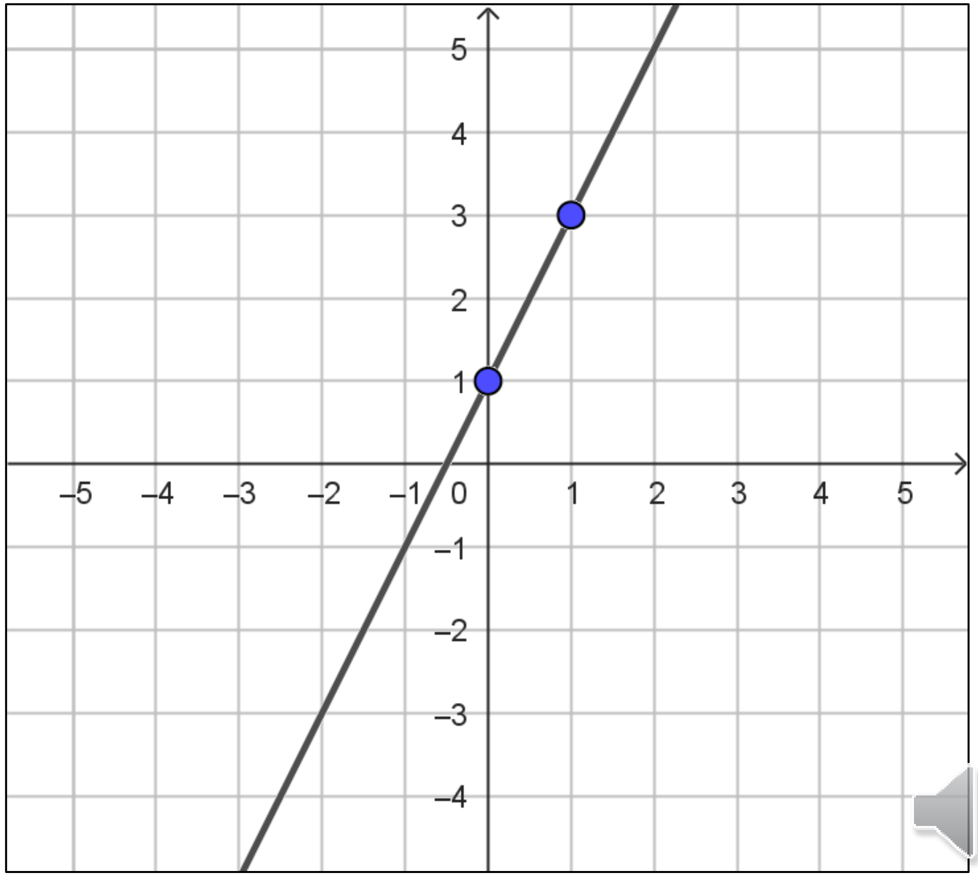
Draw the line of equation y= $2x + 1$.

For $x = 0$; $y = 2(0) + 1 = 1$

So (0;1)

For $x = 1$; $y = 2(1) + 1 = 3$

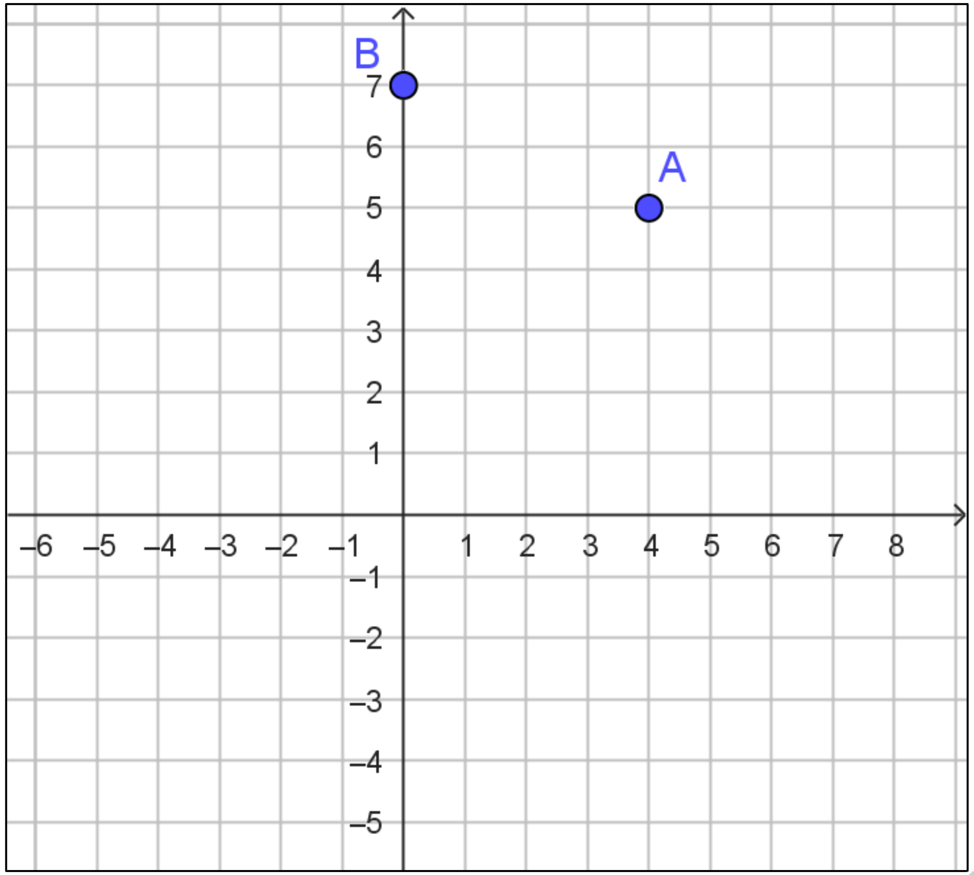
So (1;3)



☞Application #2 Session 2024 (1)

In an orthonormal system pf axes (x’Ox;y’Oy), consider the two points A(4;5) and B(0;7). Let (d) be the line with equation $y = 2x - 3$.

1) Plot the points A and B.



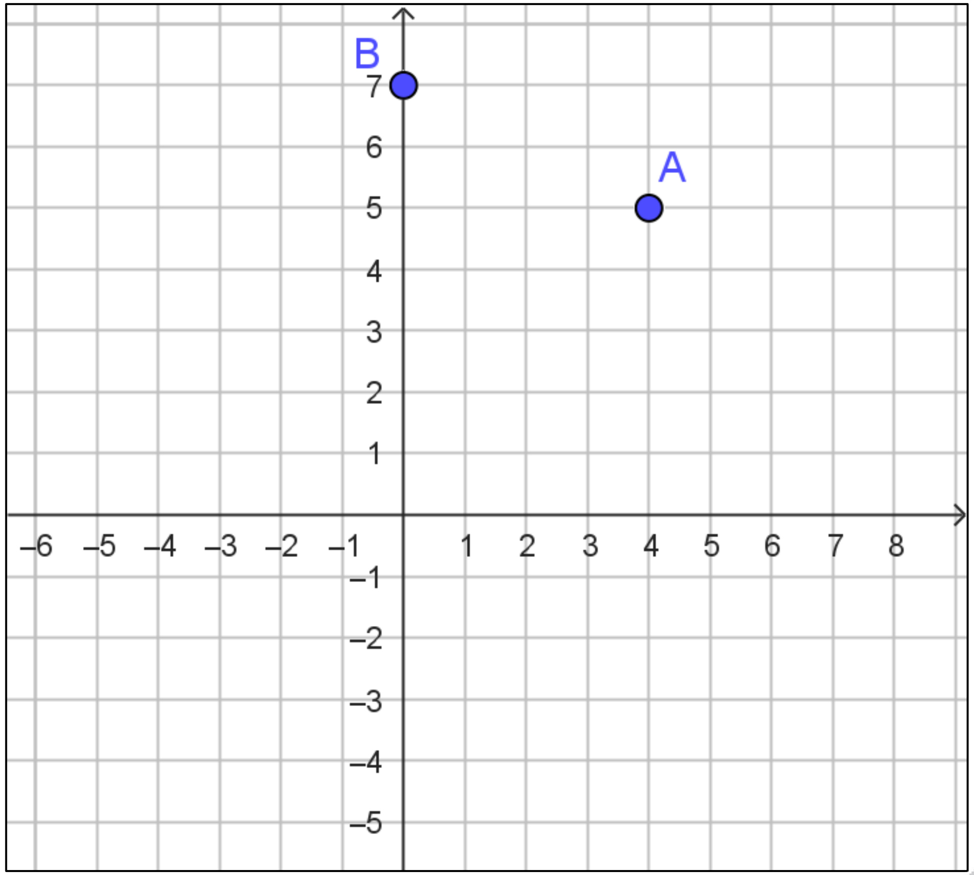
☞Application #2 Session 2024 (1)

In an orthonormal system pf axes (x’Ox;y’Oy), consider the two points A(4;5) and B(0;7). Let (d) be the line with equation $y = 2x - 3$.

2) Verify that the point A is on (d).

$$2x_A - 3 = 2(4) - 3 = 8 - 3 = 5 = y_A$$

So A is on (d)



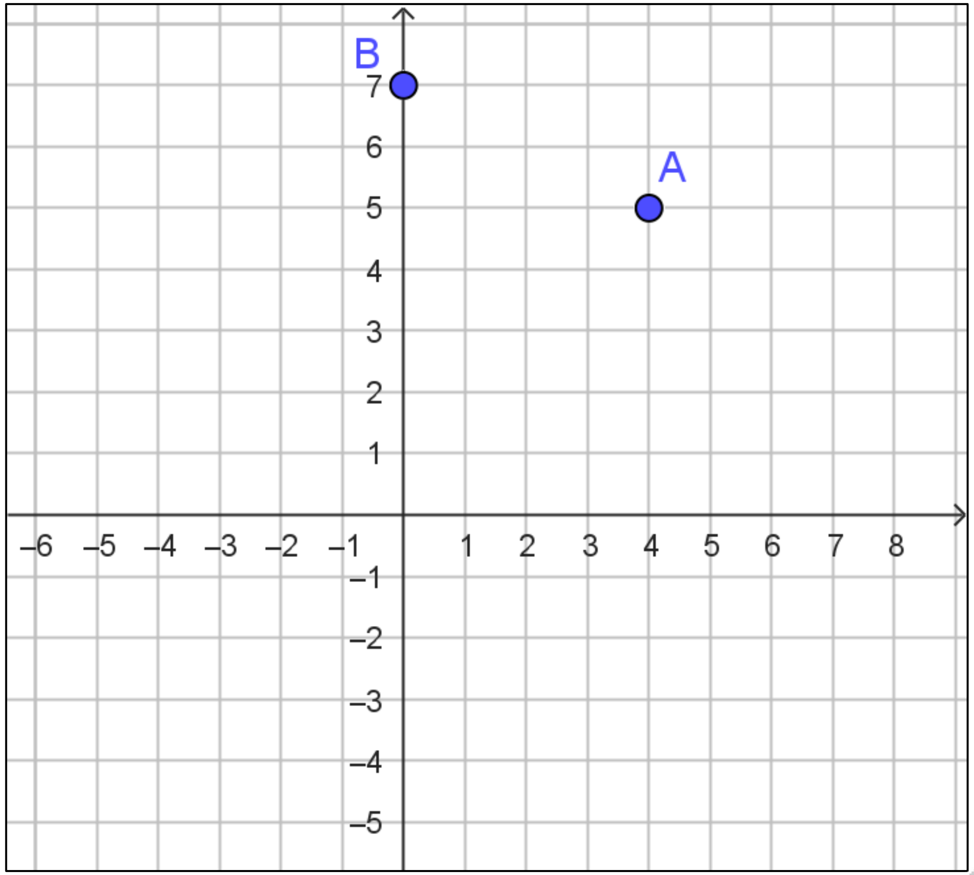
☞Application #2 Session 2024 (1)

In an orthonormal system pf axes (x'Ox;y'Oy), consider the two points A(4;5) and B(0;7). Let (d) be the line with equation $y = 2x - 3$.

3) Determine the coordinates of the point E, the intersection of (d) with (y'y).

For $x = 0$; $y = 2(0) - 3 = -3$

So E(0;-3)



☞Application #2 Session 2024 (1)

In an orthonormal system pf axes (x'Ox;y'Oy), consider the two points A(4;5) and B(0;7). Let (d) be the line with equation $y = 2x - 3$.

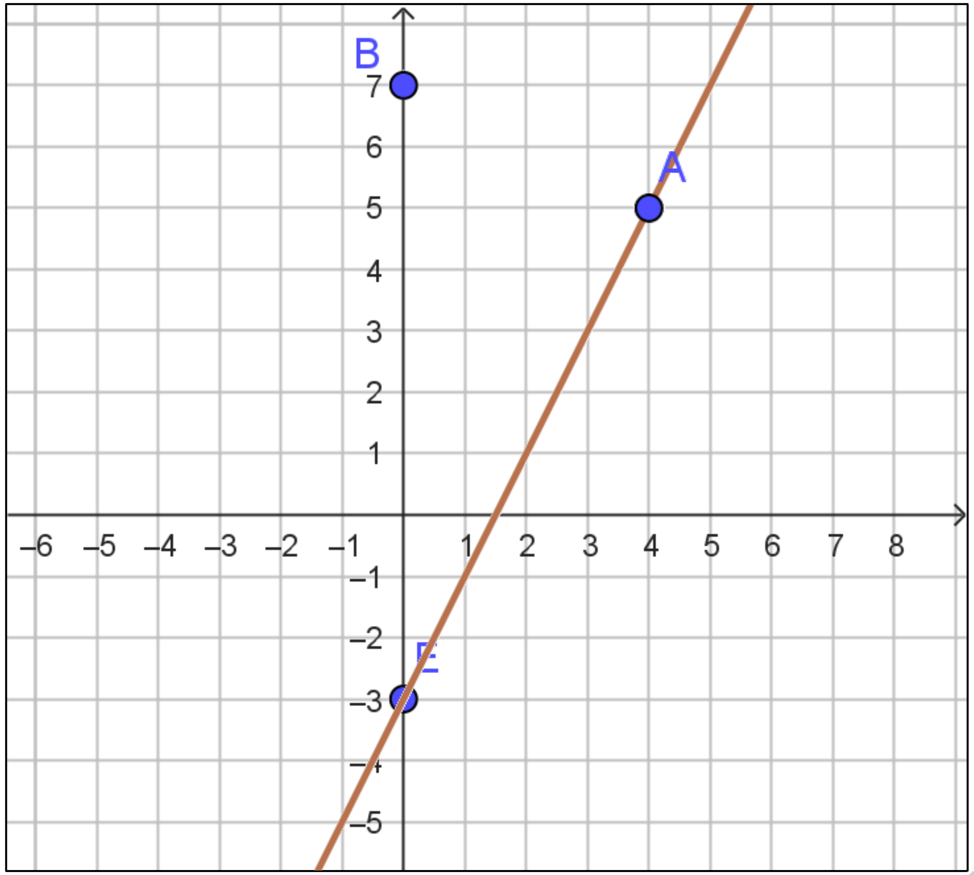
4) Draw (d).

Remark:

To draw the line (d), 2 points are needed.

First point is A.

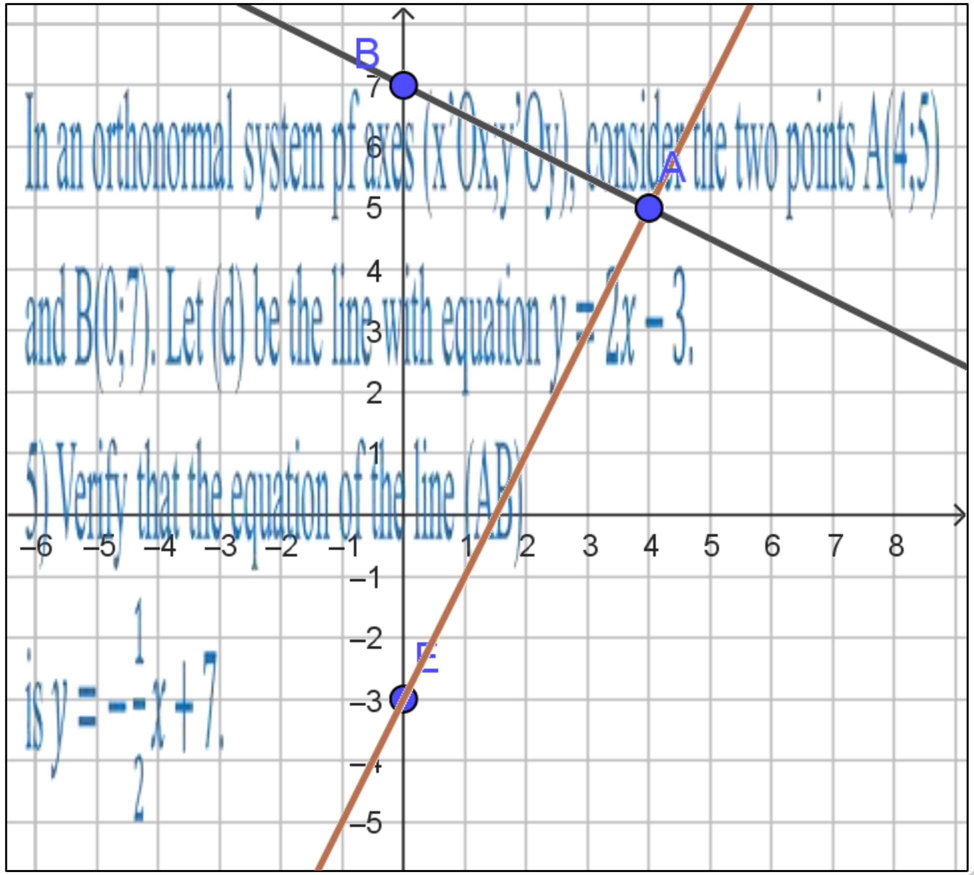
Second point is E



☞Application #2 Session 2024 (1)

In an orthonormal system pf axes (x’Ox;y’Oy), consider the two points A(4;5) and B(0;7). Let (d) be the line with equation $y = 2x - 3$.

5) Verify that the equation of the line (AB) is $y = -\frac{1}{2}x + 7$.



First method:

$$-\frac{1}{2}x_A + 7 = -\frac{1}{2}(4) + 7 = 5 = y_A$$

So the coordinates of A verify the equation.

$$-\frac{1}{2}x_B + 7 = -\frac{1}{2}(0) + 7 = 7 = y_B$$

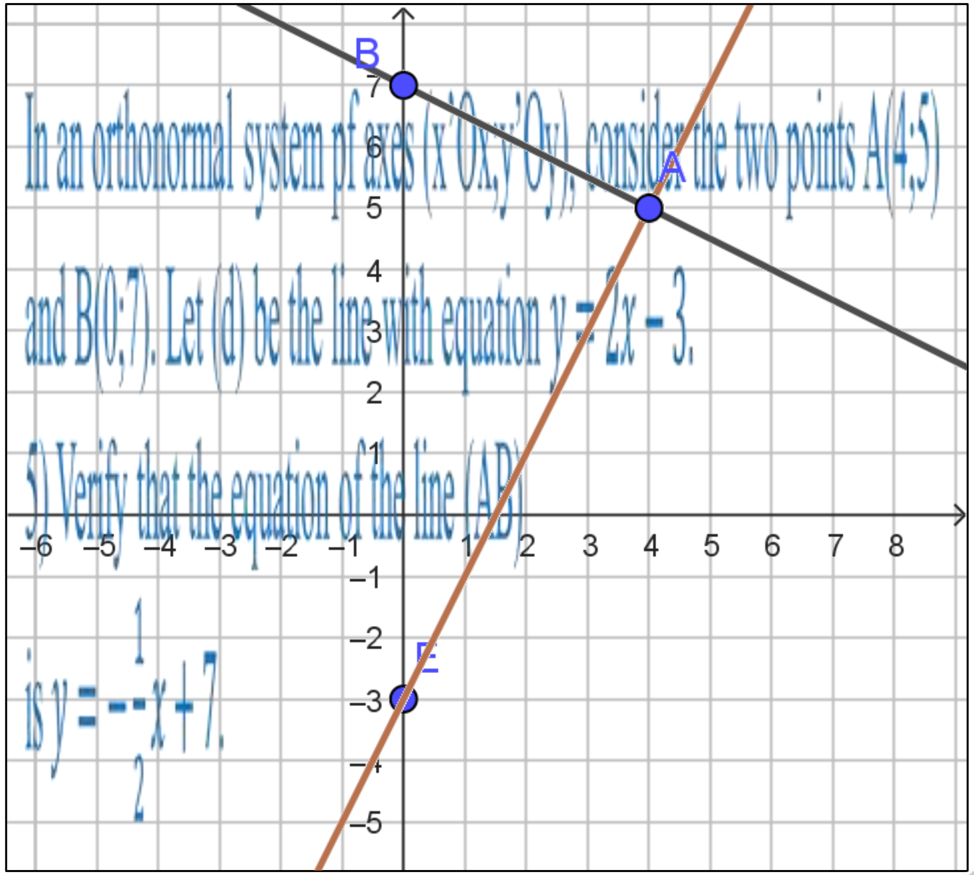
So the coordinates of B verify the equation.

Then the equation of (AB) is $y = -\frac{1}{2}x + 7$

☞Application #2 Session 2024 (1)

In an orthonormal system pf axes (x'Ox;y'Oy), consider the two points A(4;5) and B(0;7). Let (d) be the line with equation $y = 2x - 3$.

5) Verify that the equation of the line (AB) is $y = -\frac{1}{2}x + 7$.



Second method:

Slope $a = \frac{y_A - y_B}{x_A - x_B} = \frac{5-7}{4-0} = -\frac{1}{2}$

General form: $y - y_B = a(x - x_B)$

$y - 7 = -\frac{1}{2}(x - 0)$

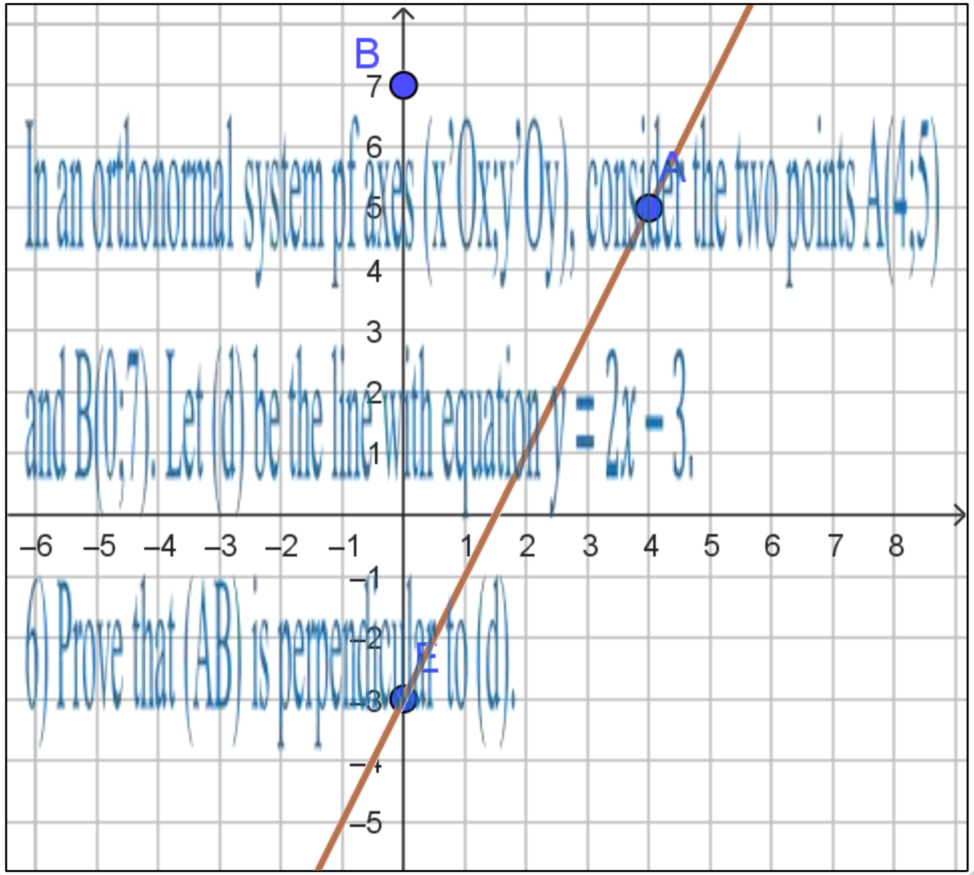
$y - 7 = -\frac{1}{2}x$

$y = -\frac{1}{2}x + 7$

☞Application #2 Session 2024 (1)

In an orthonormal system pf axes (x’Ox;y’Oy), consider the two points A(4;5) and B(0;7). Let (d) be the line with equation $y = 2x - 3$.

6) Prove that (AB) is perpendicular to (d).



$$(AB): y = -\frac{1}{2}x + 7$$

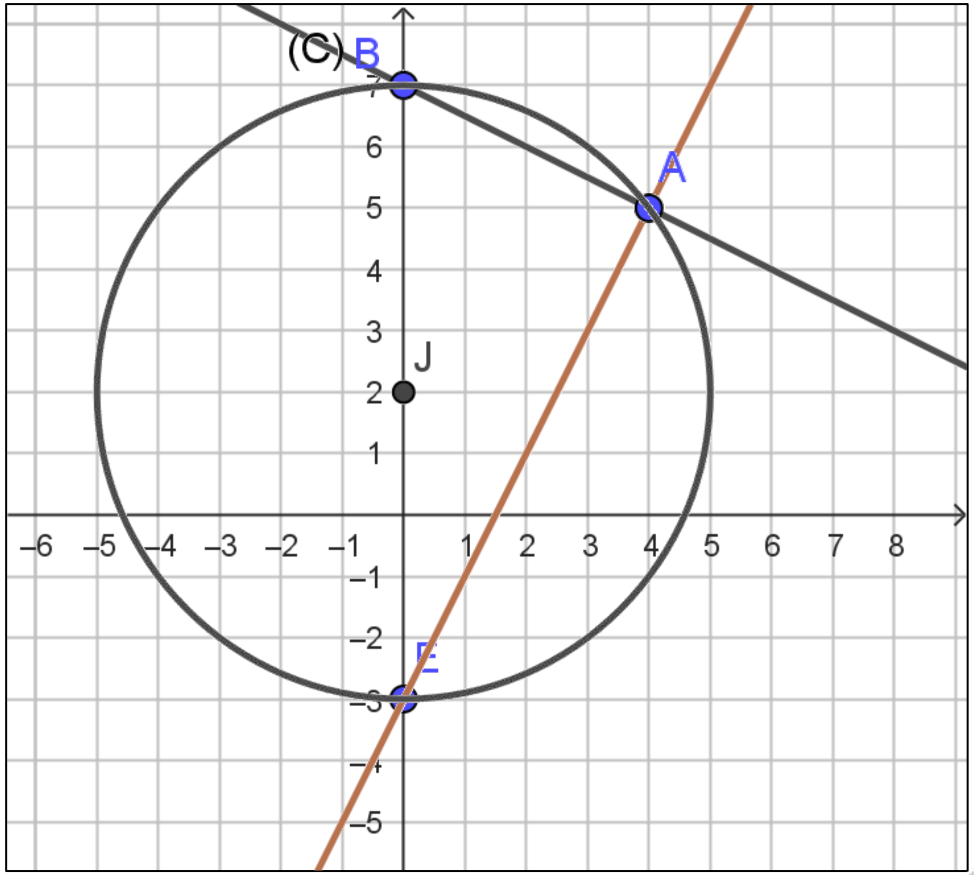
$$a_{(AB)} \times a_{(d)} = -\frac{1}{2} \times 2 = -1$$

So (AB) and (d) are perpendicular.

☞ Application #2 Session 2024 (1)

In an orthonormal system pf axes (x'Ox;y'Oy), consider the two points A(4;5) and B(0;7). Let (d) be the line with equation $y = 2x - 3$.

7) Let J be the center of the circle (C) circumscribed about the triangle ABE. Show that the coordinates of J are (0;2).



J is the midpoint of [EB]:

$$x_J = \frac{x_B + x_E}{2} = \frac{0+0}{2} = 0$$

$$y_J = \frac{y_B + y_E}{2} = \frac{7-3}{2} = 2$$

So J(0;2)

☞Application #2 Session 2024 (1)

In an orthonormal system pf axes (x’Ox;y’Oy), consider the two points A(4;5) and B(0;7). Let (d) be the line with equation $y = 2x - 3$.

8) Calculate the radius of (C).

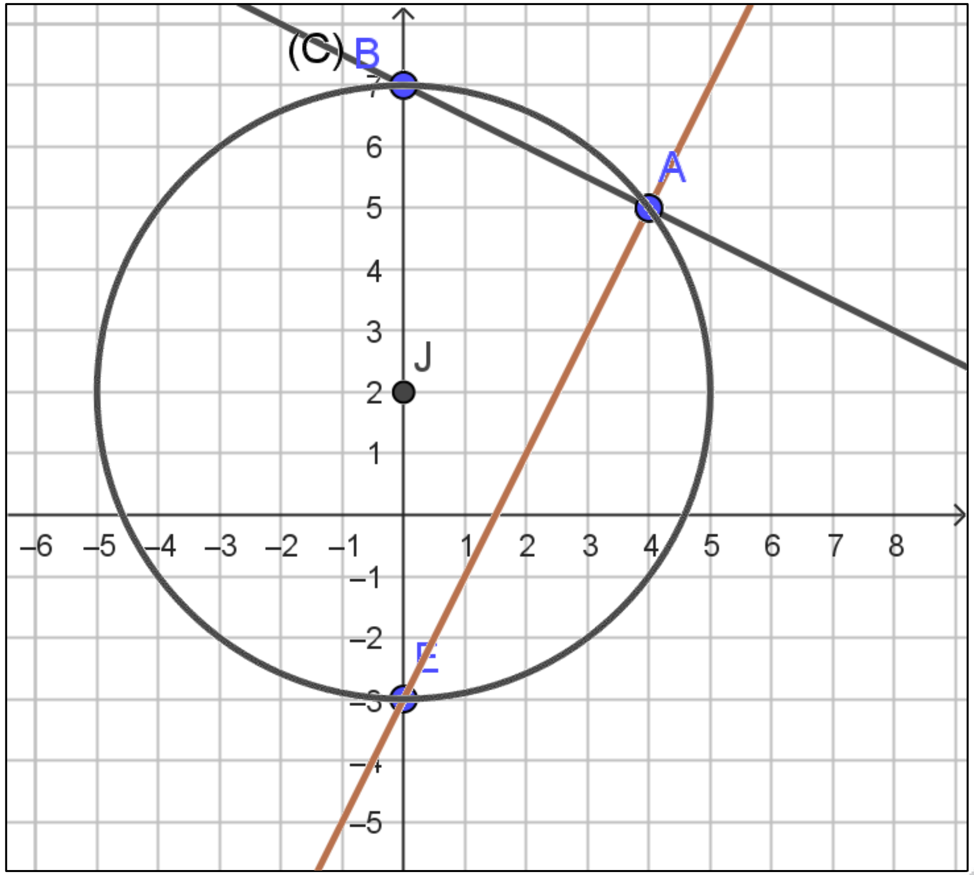
Radius R=JB

J(0;2)

$$R = \sqrt{\left(x_B - x_J\right)^2 + \left(y_B - y_J\right)^2}$$

$$= \sqrt{(0-0)^2 + (7-2)^2}$$

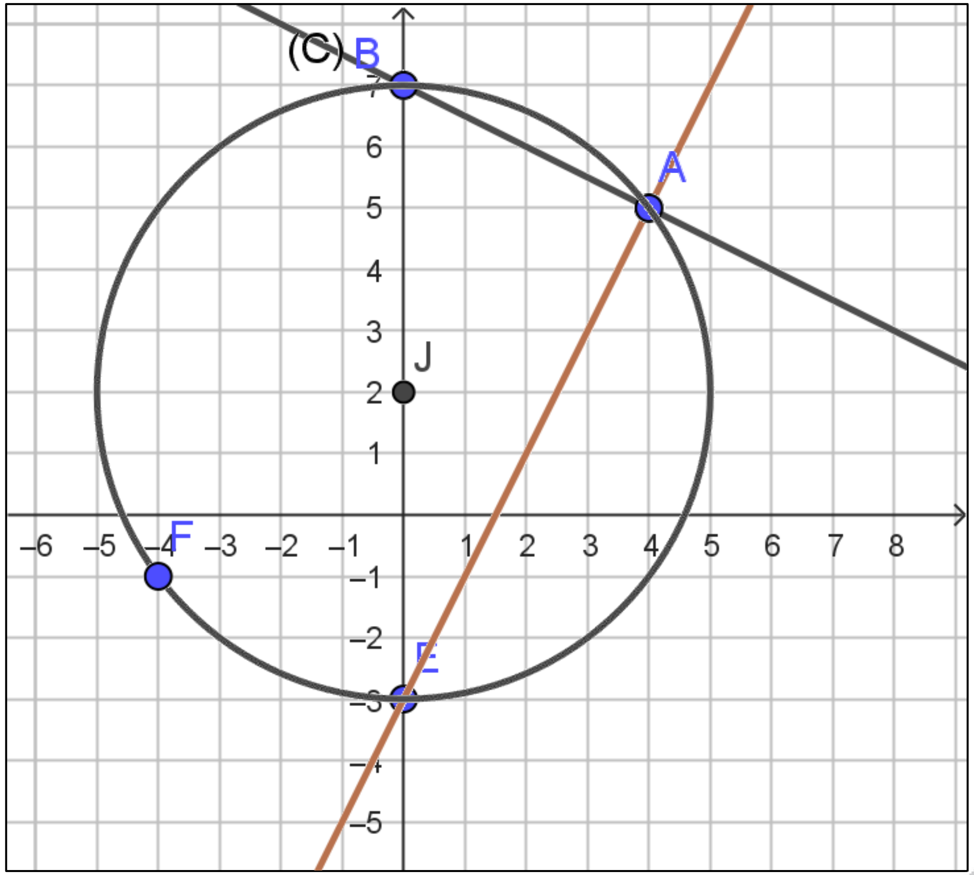
$$= 5$$



☞Application #2 Session 2024 (1)

In an orthonormal system pf axes (x'Ox;y'Oy), consider the two points A(4;5) and B(0;7). Let (d) be the line with equation $y = 2x - 3$.

9) Determine the coordinates of the point F such that the quadrilateral EABF is a rectangle.



EABF is a rectangle, so the diagonal bisect each other.

J is the midpoint of [EB] so J is the midpoint of [AF].

$x_F = 2x_J - x_A = 2(0) - 4 = -4$

$y_F = 2y_J - y_A = 2(2) - 5 = 4 - 5 = -1$

So F(-4;-1)